Introduction to Trigonometry

Assertion & Reason Type Questions

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option:

a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

b. Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

c. Assertion (A) is true but Reason (R) is false.

d. Assertion (A) is false but Reason (R) is true.

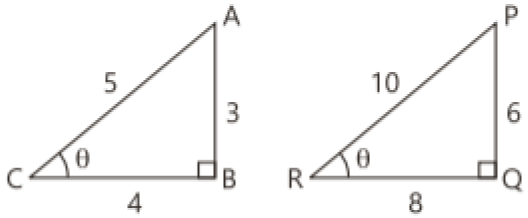
Q 1. Assertion (A): The value of each of the trigonometric ratios of an angle do not vary with the lengths of the sides of the triangle, if the angle remains the same.

Reason (R): In right angled ΔABC, <B = 90° and

$$\angle A = \theta, \quad \sin\theta = \frac{BC}{AC} < 1 \quad \text{and} \quad \cos\theta = \frac{AB}{AC} < 1 \quad \text{as}$$

hypotenuse is the longest side.

Answer : (b) **Assertion (A):** Suppose in ΔABC and in ΔPQR



$$\sin\theta = \frac{AB}{AC} = \frac{3}{5} \text{ and } \sin\theta = \frac{PQ}{PR} = \frac{6}{10}$$

$$\Rightarrow \qquad \sin\theta = \frac{3}{5} \text{ and } \sin\theta = \frac{3}{5}$$

Similarly, this will also holds for other trigonometric ratios.

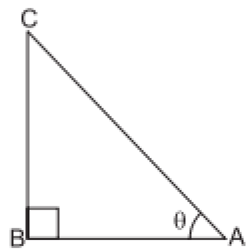
So, trigonometric ratio does not depend on the size of the triangle.

So, Assertion (A) is true.

Reason (R): Given, B = <90°, <A= θ and

$$\sin\theta = \frac{BC}{AC} < 1$$

$$\text{and } \cos\theta = \frac{AB}{AC} < 1$$



$$\therefore \quad \sin^2\theta + \cos^2\theta < 1$$

$$\Rightarrow \quad \left(\frac{BC}{AC}\right)^2 + \left(\frac{AB}{AC}\right)^2 < 1$$

$$\Rightarrow \quad \frac{BC^2}{AC^2} + \frac{AB^2}{AC^2} < 1$$

$$\Rightarrow \quad AB^2 + BC^2 < AC^2$$

So, in right ΔABC hypotenuse is the longest side.

:- Reason (R) is also true.

Hence, both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

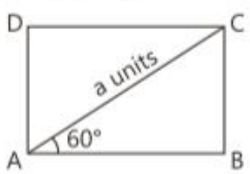
Q 2. Assertion (A): ΔBCD is a rectangle such that <CAB=60° and AC = a units. The area of rectangle

ABCD is $\frac{\sqrt{3}}{2}a^2$.

Reason (R): The value of sin 60° is $\frac{\sqrt{3}}{2}$ and cos 60° is $\frac{1}{2}$.

Answer : (d) **Assertion (A):** In ΔABC, AC = a units, /A = 60°

$$\therefore \quad \sin 60^\circ = \frac{BC}{AC} = \frac{BC}{a}$$



$$\Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{BC}{a} \quad \Rightarrow BC = \frac{a\sqrt{3}}{2}$$

$$\text{Also, } \cos 60^\circ = \frac{AB}{AC} \quad \Rightarrow \frac{1}{2} = \frac{AB}{a}$$

$$\Rightarrow \quad AB = a/2$$

$\therefore$ Area of rectangle ABCD = AB × BC

$$= \frac{a}{2} \times \frac{a\sqrt{3}}{2} = \frac{\sqrt{3}}{4}a^2$$

So, Assertion (A) is false.

Reason (R): It is true to say that the value of sin 60°

is $\frac{\sqrt{3}}{2}$ and cos 60° is $\frac{1}{2}$.

Hence, Assertion (A) is false but Reason (R) is true.

Q 3.

Assertion (A): If $\sin\theta = \frac{1}{2}$ and θ is acute angle, then $(3\cos\theta - 4\cos^3\theta)$ is equal to 0.

Reason (R): As $\sin\theta = \frac{1}{2}$ and θ is acute, so θ must be 60°.

Answer :

(c) **Assertion (A):** We have, $\sin\theta = \frac{1}{2}$

$$\Rightarrow \qquad \theta = 30^\circ \qquad \left(\because \sin 30^\circ = \frac{1}{2}\right)$$

$$\therefore \quad 3\cos\theta - 4\cos^3\theta = 3\cos 30^\circ - 4\cos^3 30^\circ$$

$$= \frac{3\sqrt{3}}{2} - 4\left(\frac{\sqrt{3}}{2}\right)^3 = \frac{3\sqrt{3}}{2} - \frac{3\sqrt{3}}{2} = 0$$

So, Assertion (A) is true.

Reason (R): It is false to say that at $\theta = 60^\circ$, $\sin\theta = \frac{1}{2}$.

This will be correct at θ = 30°.

Hence, Assertion (A) is true but Reason (R) is false.

Q 4.

Assertion (A): In a right-angled triangle, if $\tan\theta = \frac{3}{4}$,

the greatest side of the triangle is 5 units.

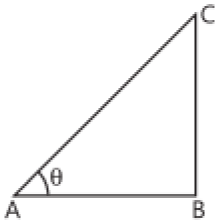
Reason (R): $(\text{Greatest side})^2 = (\text{Hypotenuse})^2$

$= (\text{Perpendicular})^2 + (\text{Base})^2$.

Answer :

(a) **Assertion (A):** Given,

$$\tan\theta = \frac{3}{4} = \frac{\text{Perpendicular}}{\text{Base}} = \frac{BC}{AB}$$



Let BC = 3k and AB = 4k

In right-angled ΔABC, by Pythagoras theorem,

$$AC = \sqrt{(AB)^2 + (BC)^2}$$
$$= \sqrt{(4k)^2 + (3k)^2} = \sqrt{16k^2 + 9k^2}$$
$$= \sqrt{25k^2} = 5k$$

It is true to say that greatest side of a triangle is hypotenuse.

So, Assertion (A) is true.

Reason (R): It is also true.

Hence, both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

Q 5. Assertion (A): For $\theta° < \theta \leq 90°$, cosec θ - cot θ and cosec θ + cot θ are reciprocal of each other.

Reason (R): $\text{cosec}^2\,\theta - \cot^2\,\theta = 1$.

Answer :

(a) **Assertion (A):** For $0° < \theta \leq 90°$, $\text{cosec}^2\theta - \cot^2\theta = 1$

$\Rightarrow$ $(\text{cosec}\,\theta - \cot\,\theta)\,(\text{cosec}\,\theta + \cot\,\theta) = 1$

$[\because a^2 - b^2 = (a-b)\,(a+b)]$

$\Rightarrow$ $\text{cosec}\,\theta - \cot\,\theta = \dfrac{1}{\text{cosec}\,\theta + \cot\theta}$

Or

$\text{cosec}\,\theta + \cot\,\theta = \dfrac{1}{\text{cosec}\,\theta - \cot\theta}$

:- (cosec e-cot 0) and (cosec + cot 0) are reciprocal of each other.

So, Assertion (A) is true.

Reason (R): It is also true.

Hence both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

Q.6. Assertion (A) : The value of $\sin 60^0 \cos 30^0 + \sin 30^0 \cos 60^0$ is 1

Reason (R) : $\sin 90^0=1$ and $\cos 90^0=0$

Answer : (b)

Q.7. Assertion (A) : The value of $2\tan^2 45^0+\cos^2 30^0-\sin^2 60^0$ is 2.

Reason (R) : value of $\tan 45^0=1$, $\cos 30^0=\sqrt{3}/2$ and $\sin 60^0=\sqrt{3}/2$.

Answer : (a)

Q.8. Assertion (A) : If $x=2\sin^2\theta$ and $y=2\cos^2\theta+1$ then the value of $x+y=3$.

Reason (R) : For any value of θ, $\sin^2\theta+\cos^2\theta=1$

Answer : (a)

Q.9. Assertion (A) : sinA is the product of sin A.

Reason (R) : The value of $\sin\theta$ increases as θ increases.

Answer : (d)

Q.10. Assertion (A) : In a right ΔABC, right angled at B, if $\tan A=1$, then $2\sin A.\cos A=1$

Reason (R) : cosecA is the abbreviation used for cosecant of angle A.

Answer : (b)

Q.11. Assertion (A) : In a right ΔABC, right angled at B, if $\tan A=12/5$, then $\sec A=13/5$.

Reason (R) : cotA is the product of cot and A.

Answer : (c)

Q12. Assertion (A) : If $x\sin^3\theta+y\cos^3\theta = \sin\theta\cos\theta$ and $x\sin\theta = y\cos\theta$, then $x^2+y^2=1$.

Reason (R) : For any value of θ, $\sin^2\theta+\cos^2\theta=1$

Answer : (a)

Q.13. Assertion (A) : $(\cos^4 A - \sin^4 A)$ is equal to $2\cos^2 A - 1$.

Reason (R) : The value of $\cos\theta$ decreases as θ increases.

Answer : (b)

Q.14. Assertion (A) : If $\cos A + \cos^2 A = 1$ then $\sin^2 A + \sin^4 A = 1$.

Reason (R) : $\sin^2 A + \cos^2 A = 1$, for any value of A.

Answer : (a)

Q.15. Assertion (A) : $\sin(A+B) = \sin A + \sin B$

Reason (R) : For any value of θ, $1 + \tan^2\theta = \sec^2\theta$

Answer : (d)